CHAPTER-1
Graph Theory

Syllabus

UNIT I Graph Theory: Pre- Requisites: Basic circuital law, Mesh & Nodal analysis. Importance of Graph Theory in Network Analysis, Graph of a network, Definitions, planar &Nonplanar Graphs, Isomorphism, Tree, Co Tree, Link, basic loop and basic cut set, Incidence matrix, Cut set matrix, Tie set matrix, Duality, Loop and Nodal methods of analysis.

Outcome

Apply the knowledge of basic circuital law, nodal and mesh methods of circuit analysis and simplify the network using Graph Theory approach.

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Unit -1 (Graph Theory)

Network Topology is a graphical representation of electric circuits. It is useful for analyzing complex electric circuits by converting them into Network graphs. Network Topology is also called as "Graph theory".

Basic Terminology used in Network Graph

1. **Node** – It is a point at which two or more Branches are connected together. It is also called as principle node.

2. **Branch** – It is a line joining two nodes. A branch always represents a circuit element in the given network.

3. **Degree of Node** – The number of branches terminating at a node is called as degree of Node.

4. **Network** – The interconnection of two or more simple circuit elements is called a network.

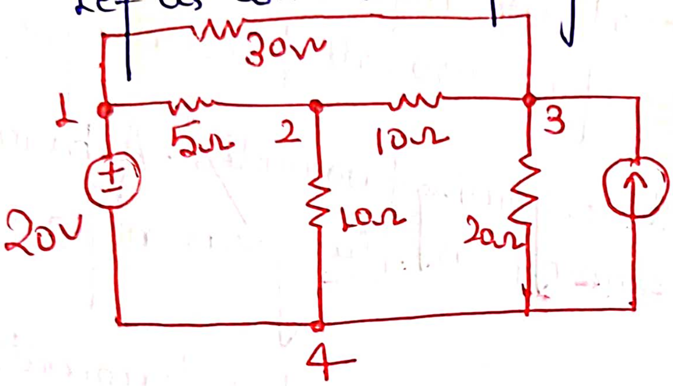
5. **Circuit** – If the n/w (network) contains at least one closed path, it is called electric circuits.

6. **Graph** – It consist of a set of nodes connected by branches. In graphs, a node is a common point of two or more branches. Sometimes only a single branch may connect to the node.

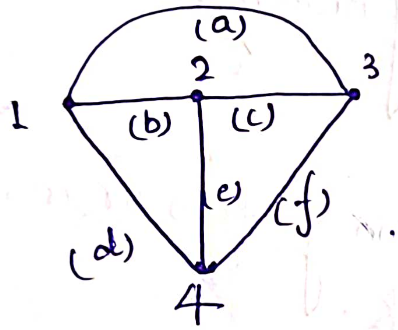
Any electric circuit or network can be converted into its equivalent Graph by replacing the passive elements and voltage source with short circuit and the current source with open circuits.

That means the line segment in the graph represents the branches corresponding to either passive elements or voltage sources of electric circuit.

Example- Let us consider the following circuit -



In the given circuit there are 4 nodes and 7 branches.

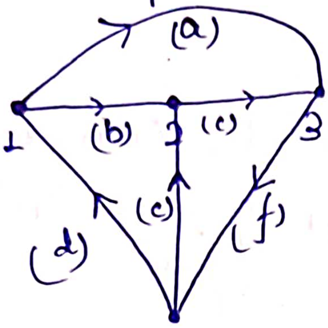


(equivalent Graph)

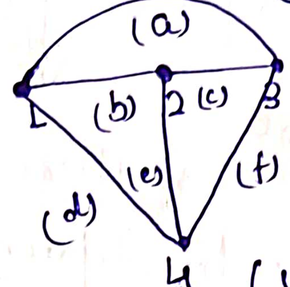
when all the elements in a network are replaced by lines with dots at both end. the configuration is then called graph.

Oriented and Unoriented Graphs –

If the Directions of current are given in the network then it is called as oriented or Directed graph. And if the Directions of current are not given in the graph then it is called as unoriented or undirected graph.



(Oriented Graph)

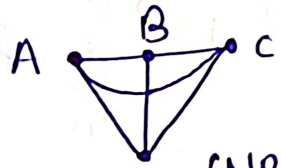


(Un-oriented Graph)

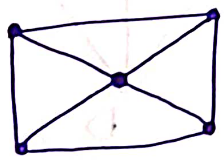
Planer and Non-Planer Graphs –

If a graph is drawn on two dimensions plane such that no two branches intersects or cross at a point that is other than a node is called a planer Graph

A non-planer Graph is also drawn on two dimensions plane such that two or more branches intersects or cross at a point other than node.



(Non planer)



(planer)

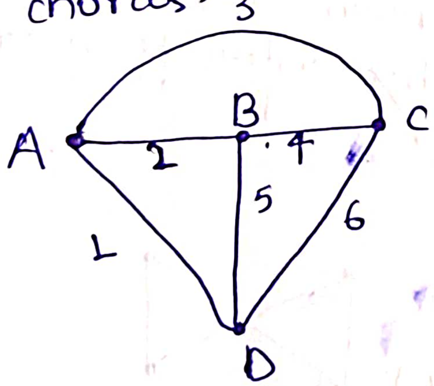
Sub-Graph - It is a subset of Branches and nodes of Graph.

* A proper sub-graph consist of Branches and nodes that are less than the original graph.
* Improper sub-graph is a graph which contains all the nodes as that of original graph.

Rank of Graph - If there are N no. of nodes then rank of Graph can be obtained from (N-1).

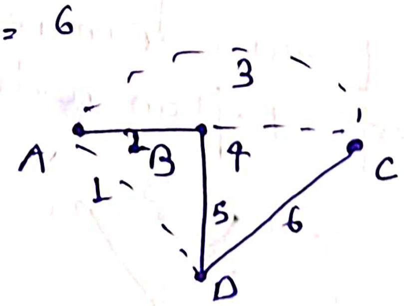
Tree - It is an interconnected open set of branches which include all the nodes of the given graph, but not containing any closed loop.

The branches of Tree is called twig. The branches which are not in the tree are called as ~~call~~ links or chords.



N = 4 (A, B, C, D)

B = 6



Twigs = 2, 5, 6

Links = 1, 3, 4

Properties of Tree -

(1) It contains all the nodes
(2) It does not contain any closed path.
(3) Rank of tree = Rank of Graph
(4) The number of Branches in a tree = No. of Nodes - 1 (N - 1)

Co-tree - A set of Branches forming complement as tree is called as co-tree.

$$L = B - N + 1$$

L = No. of links
B = No. of Branches
N = No. of Nodes.

Path - An ordered sequence of Branches transferring from one node to another is called a path in a graph.

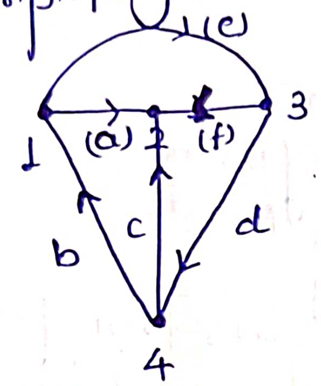
Connected Graph - If the path is existing between any two nodes in the graph then such graph is called as connected graph.

Isomorphism - It is the property between two graphs so that both have got same incidence matrix.

Incident Matrix — (Aa) (Incidence matrix)

This matrix shows which branch incident to which node. In this matrix rows of the matrix represents the number of nodes and columns of matrix represent number of Branches in the given graph.

When a graph does have N nodes and B Branches the complete incidence matrix is [N×B] rectangular matrix.

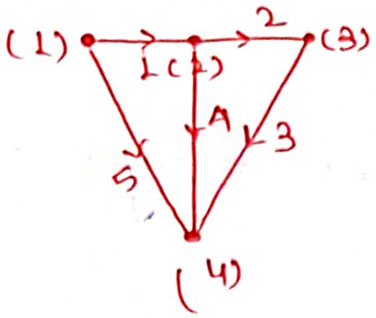


In the given graph or directed graph there are 4 nodes and 6 branches thus the incidence matrix for the above graph will have 4 rows and 6 columns.

The entries of Incidence matrix is alway -1, +1, 0
This matrix is always analogous to KCL.

Type of Branch	Value
Outgoing branch from node	+1
Incomming branch towards node	-1
others	0

(7)



Incidence matrix - Branches

A_a =

Nodes	(1)	(2)	(3)	(4)	(5)
(1)	+1	0	0	0	+1
(2)	-1	+1	0	+1	0
(3)	0	-1	+1	0	0
(4)	0	0	-1	-1	-1

Properties of Incidence matrix -

(1) Algebric sum of column entries of an Incidence matrix is zero.

(2) Determinant of the incidence matrix of a closed loop is zero.

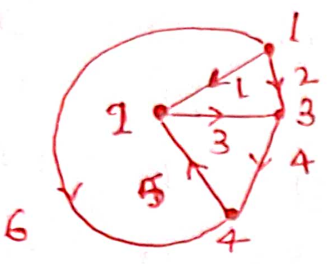
Reduced Incidence Matrix -

When one Row is deleted from complete incidence matrix. then the Remaining matrix is called Reduced Incidence matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & +1 & 0 & +1 & 0 \\ 0 & -1 & +1 & 0 & 0 \end{bmatrix}$$ from above matrix

(8)

Incidence matrix and KCL -



for the graph shown in figure -

KCL at node (1)

$$i_1 + i_2 + i_6 = 0 \quad -(1)$$

KCL at node 2 -

$$-i_1 + i_3 - i_5 = 0 \quad -(2)$$

KCL at node (3) -

$$-i_2 - i_3 + i_4 = 0 \quad -(3)$$

KCL at node (4) -

$$-i_4 + i_5 - i_6 = 0 \quad -(4)$$

In matrix form these equations can be written as -

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = 0$$

$A_a \rightarrow$ complete incidence matrix

$$\boxed{A_a I_b = 0}$$

9)

Incidence matrix and KVL -

from the graph shown in fig above (1), branch voltages can be represented as node voltages -

$V_{b1} = V_{n1} - V_{n2}$ —— (1)

$V_{b2} = V_{n1} - V_{n3}$ —— (2)

$V_{b3} = V_{n2} - V_{n3}$ —— (3)

$V_{b4} = (V_{n3} - V_{n4})$ —— (4)

$V_{b5} = (-V_{n2} + V_{n4})$ —— (5)

$V_{b6} = (V_{n1} - V_{n4})$ —— (6)

In matrix form these equations can be written as -

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_{n1} \\ V_{n2} \\ V_{n3} \\ V_{n4} \end{bmatrix} = \begin{bmatrix} V_{b1} \\ V_{b2} \\ V_{b3} \\ V_{b4} \\ V_{b5} \\ V_{b6} \end{bmatrix}$$

$$A_a^T V_n = V_b$$

Number of possible Trees of a Graph -

The number of possible trees of a graph = $\text{Det}\{[A] \times [A]^T\}$

Tie-set Matrix or Loop Incidence matrix –

A tie-set is a set of Branches contained in a loop such that each loop contains one link or chord and the remainder are free branches.
It is the matrix that is used to find the branch currents.

No. of fundamental loops = No. of links of a given tree

$$\text{Loops} = B-(N-1)$$

(1) A tree is selected in the graph.

(2) form fundamental loops with each link in the graph for entire tree.

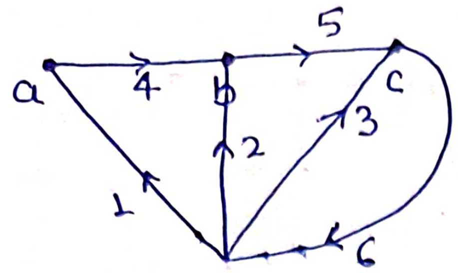
(3) Assume Direction of loop currents oriented in the same Direction as that of the link.

$B_{ij} = 1$, if the loop current and branch current flows in the same direction.

$B_{ij} = -1$, if the loop current and branch current flows in opposite direction.

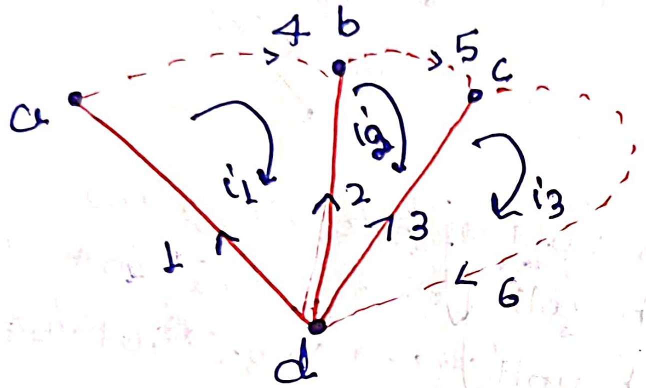
$B_{ij} = 0$ if the branch is not in the loop.

Example-



formation of loops -

(a) select a tree.



(a) Loop 1 includes Link 4 and twig 1, 2
(b) Loop 2 includes Link 5, twig 2, 3
(c) Loop 3 includes Link 6 twig 3

Tieset matrix

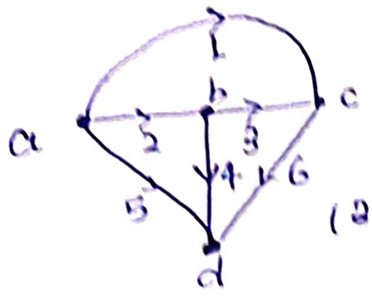
Ba =

Links(Loop)	1	2	3	4	5	6
1	1	-1	0	1	0	0
2	0	1	-1	0	1	0
3	0	0	1	0	0	1

(The heading "Branch" is written above the branch columns 1–6.)

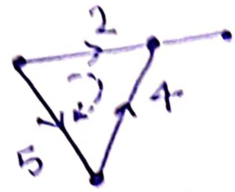
(P)

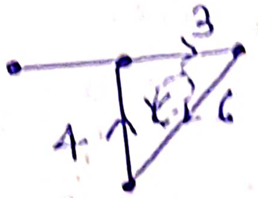
Tie-set matrix and KVL -



(3.9)

Loop(1) Loop(2) Loop(3)





for loop 1, Branch voltage → $V_{b1} - V_{b3} + V_{b2} = 0$

for loop 2, Branch voltage → $-V_{b2} - V_{b4} - V_{b5} = 0$

for loop 3, Branch voltage → $V_{b3} + V_{b6} + V_{b4} = 0$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{b1} \\ V_{b2} \\ V_{b3} \\ V_{b4} \end{bmatrix} = 0$$

$$B_a V_b = 0$$

Tie-set matrix and KCL -

for the graph shown in fig 3.9(a) and three loops shown in fig. the branch currents $i_{b1}, i_{b2}, i_{b3}, i_{b4}, i_{b5}, i_{b6}$ can be represented in terms of loop currents.

$$i'_{b1} = I'_{L1} \quad \text{--- (1)}$$

$$i'_{b2} = (I_{L1} - I_{L2}) \quad \text{--- (2)}$$

$$i'_{b3} = (-I_{L1} + I_{L3}) \quad \text{--- (3)}$$

$$i'_{b4} = (-I_{L2} + I_{L3}) \quad \text{--- (4)}$$

$$i'_{b5} = I_{L2} \quad \text{--- (5)}$$

$$i'_{b6} = I_{L3} \quad \text{--- (6)}$$

In matrix form these equations can be written as -

$$\begin{bmatrix} i'_{b1} \\ i'_{b2} \\ i'_{b3} \\ i'_{b4} \\ i'_{b5} \\ i'_{b6} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{L1} \\ I_{L2} \\ I_{L3} \end{bmatrix}$$

$$\boxed{I_b = Ba^T I_L}$$

Cut-set matrix

A cut-set ~~elements~~ is a minimum set of elements that When cut, or removed separates the graph into two groups of nodes. A cut-set is a minimum set of Branches of a connected graph, such that the removal of these branches from the graph reduces the rank of graph by one.

A fundamental cutset is a cutset that contains one and only one branch of the network tree, together with any links which must be cut to divide the n/w into two parts.

A cutset matrix is defined as a rectangular matrix whose rows corresponds to cut-sets and columns corresponds to the branches of the graph.

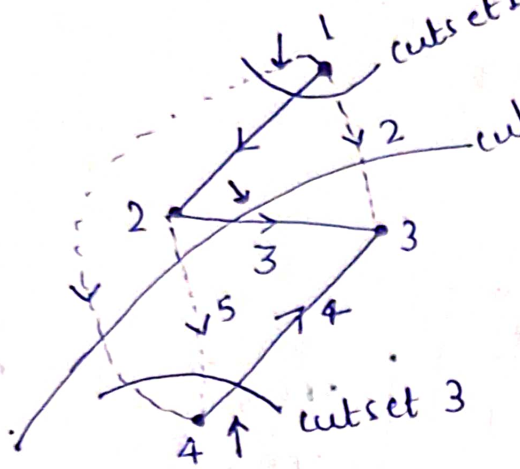
$Q_{ij} = 1$ if the branch is in the cutset and orientation coincide

$Q_{ij} = -1$ if the branch is not in the cutset and orientations do not coincide

$Q_{ij} = 0$ if the branch is not in the cut-set.

(15)

Example -



cutset-1 = {1, 2, 6}

cutset-2 = {2, 3, 5, 6}

cutset-3 = {4, 5, 6}

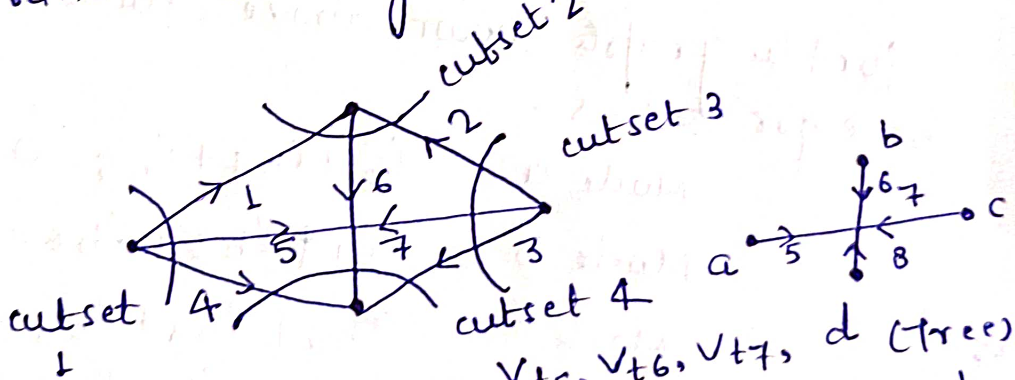
cutset matrix =

Q_c =

cutset	Branches 1	2	3	4	5	6
1	1	1	0	0	0	1
2	0	1	1	0	1	1
3	0	0	0	1	-1	-1

cutset matrix and KVL -

Branch voltages can be expressed in terms of the tree-branch voltages.



d (Tree)

tree-branch voltages are V_{t5}, V_{t6}, V_{t7}, V_{t8}. the branch voltages are $(V_{b1}, V_{b2} --- V_{b8})$.

(16)

$V_{b1} = -V_{t5} + V_{t6}$

$V_{b2} = -V_{t6} + V_{t7}$

$V_{b3} = V_{t7} - V_{t8}$

$V_{b4} = V_{t5} - V_{t8}$

$V_{b5} = V_{t5}$

$V_{b6} = V_{t6}$

$V_{b7} = V_{t7}$

$V_{b8} = V_{t8}$

$$\begin{bmatrix} V_{b1} \\ V_{b2} \\ V_{b3} \\ V_{b4} \\ V_{b5} \\ V_{b6} \\ V_{b7} \\ V_{b8} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{t5} \\ V_{t6} \\ V_{t7} \\ V_{t8} \end{bmatrix}$$

$$\boxed{V_b = Q_c^t V_t}$$

Cut-set matrix and KCL -

for the graph shown above, the branch currents can be expressed as -

Node a = $-i'_{b1} + i'_{b4} + i'_{b5} = 0$

Node b = $i'_{b1} + i'_{b2} + i'_{b6} = 0$

Node c = $i'_{b2} + i'_{b3} + i'_{b7} = 0$

Node d = $-i'_{b3} - i'_{b4} + i'_{b8} = 0$

(17)

In matrix form they can be written as -

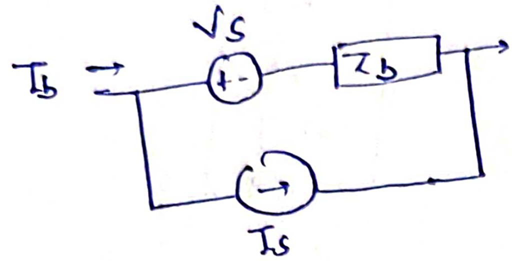
$$\begin{bmatrix} -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{b1} \\ i_{b2} \\ i_{b3} \\ i_{b4} \\ i_{b5} \\ i_{b6} \\ i_{b7} \\ i_{b8} \end{bmatrix} = 0$$

$$Q_c I_b = 0$$

Matrix	KCL	KVL
A_a (I.M)	$A_a \times I_b = 0$	$V_b = A_a^T \times V_n$
B_a (Tie-set)	$I_b = B_a^T \times I_L$	$B_a \times V_b = 0$
cut-set (Q_c)	$Q_c \times I_b = 0$	$V_b = Q_c^T \times V_t$

Node Analysis and Mesh analysis -

A general branch consisting of a voltage source V_s and a current source I_s is shown in figure.



the branch current is $(I_b + I_s)$ and the branch voltage is $(V_b + V_s)$ without sources, the KCL and KVL equations are -

$A_a \times I_b = 0$ —— (1)
$I_b = B_a^T \times I_L$ —— (2)
$Q_c \times I_b = 0$ —— (3)
} KCL

$V_b = A_a^T \times V_n$ —— (4)
$B_a \times V_b = 0$ —— (5)
$V_b = Q_c^T \times V_t$ —— (6)
} KVL

with the sources, the KCL and KVL equations are modified as -

$A_a I_b + A_a I_s = 0$ —— (7)

$I_b + I_s = B_a^T I_L$ —— (8)

$Q_c I_b + Q_c I_s = 0$ —— (9)

$V_b + V_s = A_a^T \times V_n$ —— (10)

$B_a V_b + B_a V_s = 0$ —— (11)

$V_b + V_s = Q_c^T V_t$ —— (12)

the branch voltage-current relations for the passive network elements are written in the matrix form as -

$V_b = Z_b I_b$ —— (13)

$I_b = Y_b V_b$ — (14) equ^n

Z_b = Branch Impedence matrix

Y_b = Branch admittance matrix

Node ~~fx~~ equation from equ^n (7) —

$A_a I_s = -A_a I_b$ $\{I_b = Y_b V_b\}$

$= -A_a Y_b V_b$ $\{V_b = A_a^T V_n - V_s\}$

$= -A_a Y_b \{A_a^T V_n - V_s\}$

$= -A_a Y_b A_a^T V_n + A_a Y_b V_s$

$A_a Y_b A_a^T V_n = A_a Y_b V_s - A_a I_s$

$$\boxed{Y V_n = A_a [Y_b V_s - I_s]}$$

$Y = A_a Y_b A_a^T$ is called nodal admittance matrix.

$$\boxed{Y V_n = A [Y_b V_s - I_s]}$$ {reduced matrix A}

Mesh Equation -

from equ^n (11)

$B_a V_s = -B_a V_b = -B_a (I_b Z_b)$

$= -B_a Z_b \{B_a^T I_l - I_s\}$

$= -B_a Z_b B_a^T I_l + B_a Z_b I_s$

(22)

$$B_a Z_b B_a^T I_L = B_a [Z_b I_s - V_s]$$

$$\boxed{Z I_L = B_a [Z_b I_s - V_s]}$$

$Z = B_a Z_b B_a^T$ is the loop impedance matrix

Cut-set equations -

from equation (8)

$$Q_c I_s = -Q_c I_b = -Q_c V_b Y_b$$

$$= -Q_c Y_b \{Q_c^T V_t - V_s\}$$

$$Q_c Y_b Q_c^T V_t = Q_c [Y_b V_s - I_s]$$

$$\boxed{Y_c V_t = Q_c [Y_b V_s - I_s]}$$

$Y_c = Q_c Y_b Q_c^T$ is the cutset admittance matrix

Duality of Network —

Two networks are said to be dual of each other, if the mesh equations of one are the same as the node equations of the other.

Circuit where the cut-set matrix of one corresponds to tie-set matrix of the other duals of each other.

Conversion of Dual electric circuit -

Loop Basis	Node Basis
Current	Voltage
Resistance	conductance
Inductance	capacitance
Branch current	Branch voltage
Mesh	Node
Impedance	Admittance
Voltage Division	current Division
Mesh current	Node voltage

* Put a dot per mesh
* place an extra node outside the n/w this will be the reference node.
* Draw Dotted line b/w the nodes in such a way that each line crosses only one network element and dotted line has been drawn

through every element in the original n/w.

* The Dotted line along with the dual n/w elements constitute the Dual of the original network.

Problem 3.1 ***Draw the graph of the network shown in Fig. 3.15 (a)***

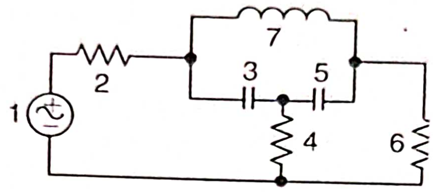


Fig. 3.15 (a)

Solution The graph of the network is shown below.

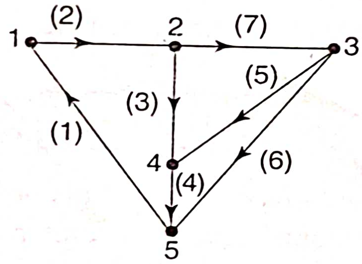


Fig. 315 (b)

Problem 3.2 ***From Fig. 3.16, make the graph and find one tree. How many mesh currents are required for solving the network? Find the number of possible trees.***

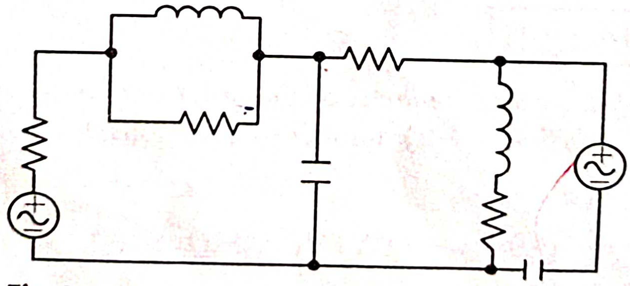


Fig. 3.16

Solution The graph of the network is shown below. One tree of the graph is shown.

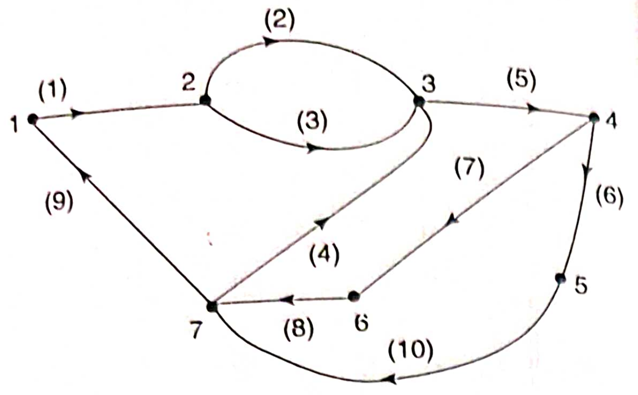


Fig. 3.17 (a) *Graph of the network*

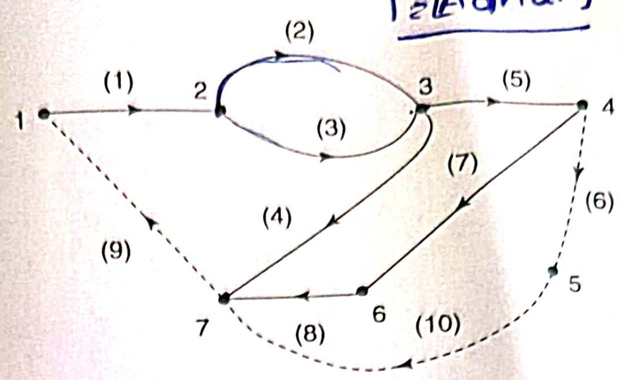


Fig. 3.17 (b) *Tree of the graph*

The complete incidence matrix is obtained as

$A_a =$

	Branches									
Nodes	1	2	3	4	5	6	7	8	9	10
1	1	0	0	0	0	0	0	0	−1	0
2	−1	1	1	0	0	0	0	0	0	0
3	0	−1	−1	1	1	0	0	0	0	0
4	0	0	0	0	−1	1	1	0	0	0
5	0	0	0	0	0	−1	0	0	0	1
6	0	0	0	0	0	0	−1	1	0	0
7	0	0	0	−1	0	0	0	−1	1	−1

The reduced incidence matrix becomes

$A =$

	Branches									
Nodes	1	2	3	4	5	6	7	8	9	10
1	1	0	0	0	0	0	0	0	−1	0
2	−1	1	1	0	0	0	0	0	0	0
3	0	−1	−1	1	1	0	0	0	0	0
4	0	0	0	0	−1	1	1	0	0	0
5	0	0	0	0	0	−1	0	0	0	1
6	0	0	0	0	0	0	−1	1	0	0

Hence the number of possible trees is

$$n = \det\left\{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}\right\}$$

$$= \det\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -2 & 0 & 0 & 0 \\ 0 & -2 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{bmatrix} \Rightarrow n = 12$$

Problem 3.3 **Branch current and loop current relations are expressed in matrix form as,**

$I_b = B_a^T \; I_L$ ✓

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

Draw the oriented graph.

Solution We know that, $[I_b] = [B_a]^T [I_L]$. So, the tie-set matrix, here, is

$B_a =$

Loop or Link Currents	Branches							
	1	2	3	4	5	6	7	8
1	1	0	0	0	1	0	−1	0
2	0	1	1	1	−1	0	0	0
3	0	0	1	1	0	−1	0	0
4	−1	−1	0	0	0	0	0	1

So, the graph consists of four loops and eight branches. Loop1 consists of branches 1, 5 and 7. The orientations are given following the sign +1 or −1. Following the procedure, the complete oriented graph is shown below.

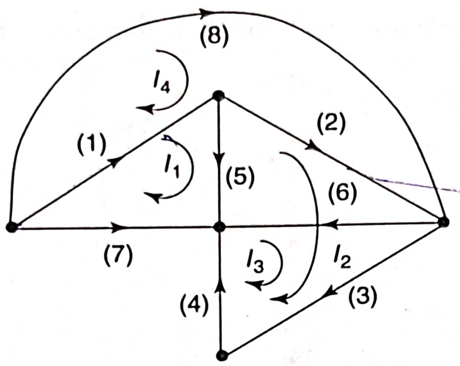


Fig. 3.18

Problem 3.4 ***The fundamental cut-set matrix is given as***

Twigs				Links		
1	2	3	4	5	6	7
1	0	0	0	−1	0	0
0	1	0	0	1	0	1
0	0	1	0	0	1	1
0	0	0	1	0	1	0

Draw the oriented graph of the network.

Solution The graph has seven branches and three fundamental cut-sets:

Cut-set-1: [1, 5]

Cut-set-2: [2, 5, 7]

Cut-set-3: [3, 6, 7]

Cut-set-4: [4, 6]

So, the oriented graph is as shown in Fig. 3.19 (a), (b), (c).

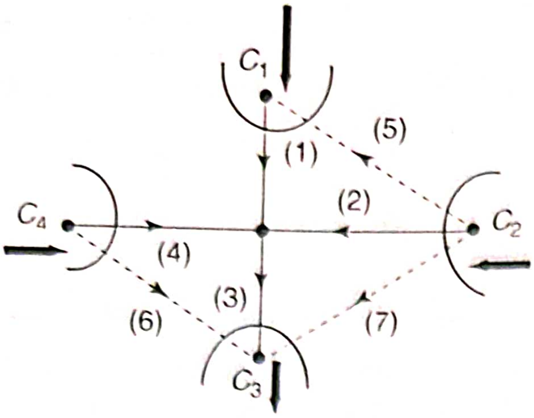


Fig. 3.19 (a)

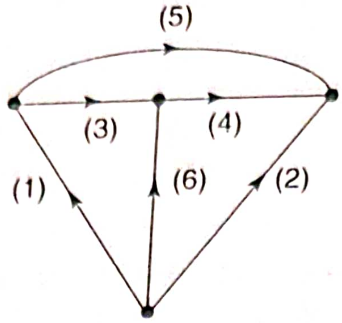


Fig. 3.19 (b)

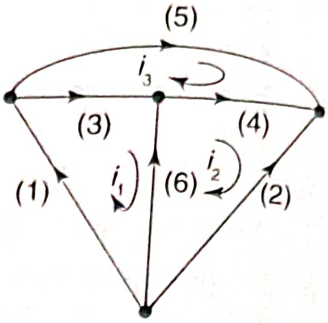


Fig. 3.19 (c)

Problem 3.5 ***Write the complete incidence matrix for the graph shown in Fig. 3.20 (a).***

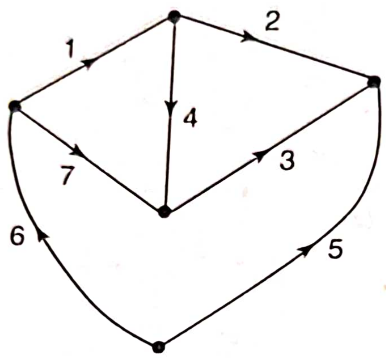


Fig. 3.20 (a)

Solution We first label the nodes as shown in Fig. 3.20 (b)

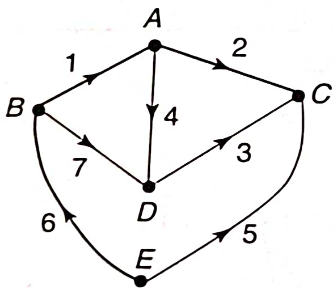


Fig. 3.20 (b)

The complete incidence matrix is given as

$A_a =$

	1	2	3	4	5	6	7
A	−1	1	0	1	0	0	0
B	1	0	0	0	0	1	1
C	0	−1	−1	0	−1	0	0
D	0	0	1	−1	0	0	−1
E	0	0	0	0	1	1	0

Problem 3.6 **Write down the incidence matrix and cut-set matrices for the network shown.**

Solution The graph and a suitable tree for the network are shown in Fig.3.21 (b).

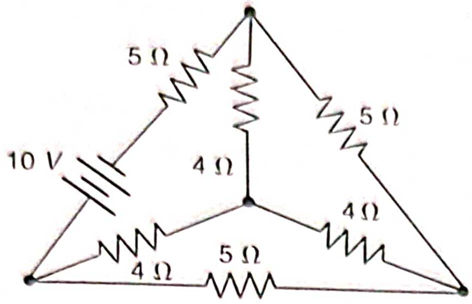


Fig. 3.21 (a)

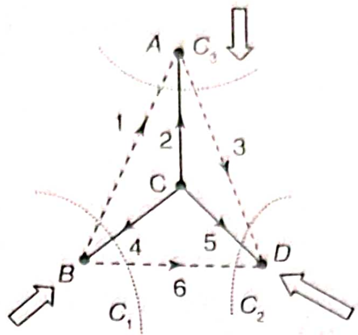


Fig. 3.21 (b)

The complete incidence matrix is given as

$A_a =$

	1	2	3	4	5	6
A	−1	−1	1	0	0	0
B	1	0	0	1	0	1
C	0	1	0	−1	1	0
D	0	0	−1	0	−1	−1

The fundamental cut-sets are identified as

f-cutset-1: [1, 4, 6]
f-cutset-2: [3, 5, 6]
f-cutset-3: ~~[1, 2, 3]~~

The fundamental cutset matrix is given as

$Q =$

	1	2	3	4	5	6
C_1	−1	0	0	1	0	1
C_2	0	0	−1	0	−1	−1
C_3	−1	−1	1	0	0	0

Problem 3.7 ***For the network shown in Fig. 3.22 (a), give fundamental cut-set matrix and hence find KCL equations.***

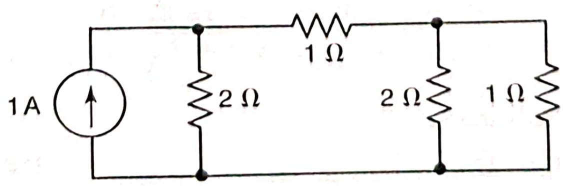


Fig.3.22 (a)

Solution The graph and one tree are shown for the network.

The fundamental cutsets are identified as

f-cutset-1: [1, 2]

f-cutset-2: [2, 3, 4]

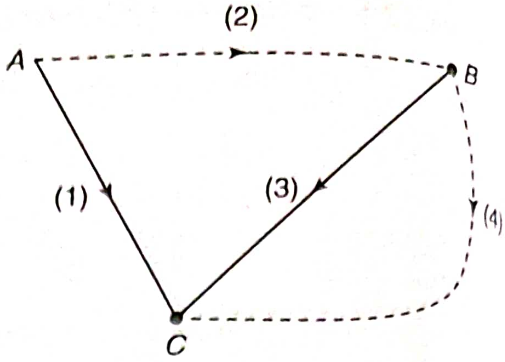


Fig. 3.22 (b)

The fundamental cut-set matrix is given as

		1	2	3	4
$Q_a =$	C_1	1	1	0	0
	C_2	0	−1	1	1

The KCL equations in terms of cut-set matrix is given as

$$[Q]\,[Y_b][Q^T][V_t] = -\,[Q]\,[I_S]$$

Here,

$$[Q][Y_b][Q^T]=\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & -1 & 1 & 1\end{bmatrix}\begin{bmatrix}2 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & -1\\ 0 & 1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}2 & 1 & 0 & 0\\ 0 & -1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & -1\\ 0 & 1\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3 & -1\\ -1 & 4\end{bmatrix}$$

$$-[Q][I_S]=-\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & -1 & 1 & 1\end{bmatrix}\begin{bmatrix}-1\\ 0\\ 0\\ 0\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$$

Thus, the KCL equations are

$$\begin{bmatrix}3 & -1\\ -1 & 4\end{bmatrix}\begin{bmatrix}V_{t1}\\ V_{t3}\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$$

Problem 3.8 ***For the network shown in Fig. 3.23 (a), draw the oriented graph, select a suitable tree and obtain the fundamental cut-set matrix. Determine the node equations and find v.***

Solution The oriented graph of the network is shown in Fig. 3.23 (b). Since we have to find v, we take the branch (2) in the twig and a possible tree is selected.

The fundamental cutsets are identified as

f-cut-set-1: [1, 2, 3]

f-cut-set-2: [3, 4]

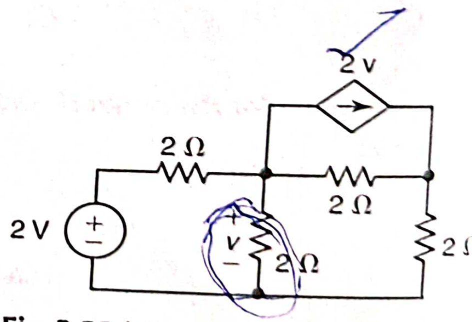


Fig. 3.23 (a)

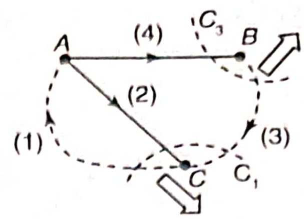


Fig. 3.23 (b)

The fundamental cut-set matrix is given as

		1	2	3	4
$Q_a =$	C_1	−1	1	1	0
	C_2	0	0	−1	1

The node equations are given as

$$[Q][Y_b][Q^T][V_t] = [Q] = \{[Y_b][V_s] - [I_s]\}$$

Here,

$$[Q][Y_b][Q^T]=\begin{bmatrix}-1 & 1 & 1 & 0\\ 0 & 0 & -1 & 1\end{bmatrix}\begin{bmatrix}1/2 & 0 & 0 & 0\\ 0 & 1/2 & 0 & 0\\ 0 & 0 & 1/2 & 0\\ 0 & 0 & 0 & 1/2\end{bmatrix}\begin{bmatrix}-1 & 0\\ 1 & 0\\ 1 & -1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}3/2 & -1/2\\ -1/2 & 1\end{bmatrix}$$

$$[Q]\times\{[Y_b][V_s]-[I_s]\}=\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & -1 & 1 & 1\end{bmatrix}\left\{\begin{bmatrix}1/2 & 0 & 0 & 0\\ 0 & 1/2 & 0 & 0\\ 0 & 0 & 1/2 & 0\\ 0 & 0 & 0 & 1/2\end{bmatrix}\begin{bmatrix}2\\0\\0\\0\end{bmatrix}-\begin{bmatrix}0\\0\\0\\2v\end{bmatrix}\right\}=\begin{bmatrix}1\\ -2v\end{bmatrix}$$

Thus, the KCL equations are

$$\begin{bmatrix}3/2 & -1/2\\ -1/2 & 1\end{bmatrix}\begin{bmatrix}V_{t2}\\ V_{t4}\end{bmatrix}=\begin{bmatrix}1\\ -2v\end{bmatrix}$$

Here, $V_{t2} = v$. Putting this in the KCL equations and solving we get, $v = \frac{4}{9}$ V

Problem 3.9 ***For the resistive network, write a cut-set schedule and equilibrium equations on voltage basis. Hence obtain values of branch voltages and branch currents.***

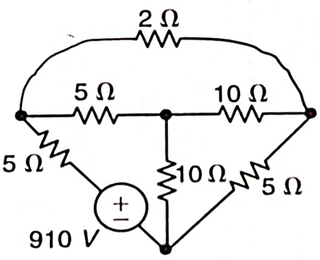


Fig. 3.24

Problem 3.11 ***For the network of Fig. 3.27, draw the graph and write a tie-set schedule. Using the tie-set schedule obtain the loop equations and find the currents in all branches.***

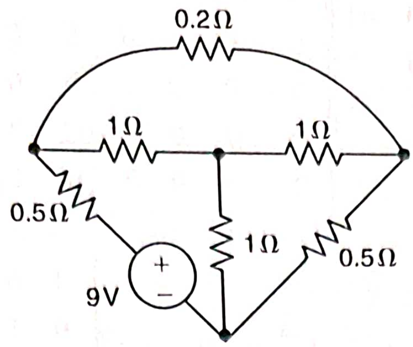


Fig. 3.27

Solution The graph and one tree are shown in Fig. 3.28.

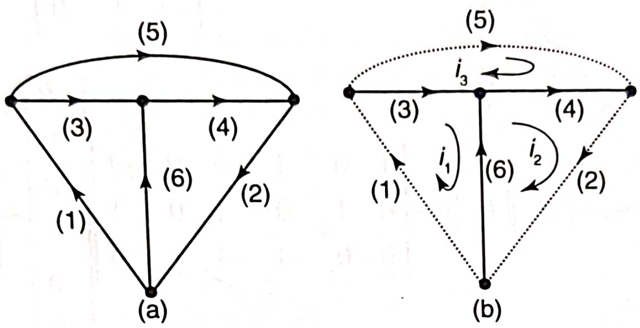


Fig. 3.28

The tie-set matrix

$$B_a = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{bmatrix}$$

Branch impedance matrix is

$$Z_b = \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$[B_a][Z_b] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0.5 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0.2 & 0 \end{bmatrix}$$

$$\therefore [B_a][Z_b][B_a]^T = \begin{bmatrix} 0.5 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0.5 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0.2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2.5 & -1 & -1 \\ -1 & 2.5 & -1 \\ -1 & -1 & 2.2 \end{bmatrix}$$

$$\text{Now, } -[B_a][V_s] = -\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -9 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$$

So, the loop equations are

$$\begin{bmatrix} 2.5 & -1 & -1 \\ -1 & 2.5 & -1 \\ -1 & -1 & 2.2 \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$$

Solving the three equations,

$$i_1 = 8.9\text{ A},\quad i_2 = 6.33\text{ A},\quad i_3 = 6.92\text{ A}$$

Problem 3.12 ***Figure 3.29 (a) shows a dc network. (a) Draw a graph of the network. Which elements are not included in the graph and why? (b) Write a loop incidence matrix and use it to obtain loop equations. (c) Find branch currents.***

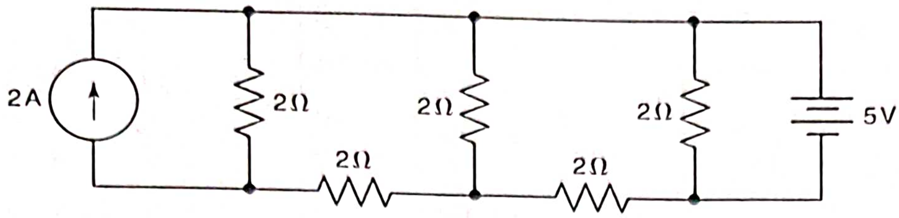


Fig. 3.29 (a)

Solution (a) The graph is shown below.

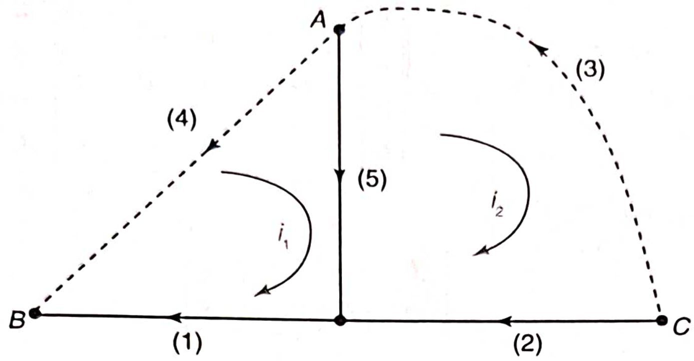


Fig. 3.29 (b)

The 2-V resistor in parallel with the voltage source and the 2-A current source have not been included in the graph. This is because of the reason that passive elements in parallel with a voltage source are not included in a graph and the current source in parallel with a passive element is open-circuited while drawing a graph.

(b) The tie-set matrix for the tree chosen is

$$B_a = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & -1 \end{bmatrix}$$

Branch impedance matrix is

$$Z_b = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$B_a Z_b B_a^T = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 & -2 & 2 \\ 0 & 2 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix}$$

Now,

$$B_a Z_b I_s - B_a V_s = \begin{bmatrix} 2 & 0 & 0 & -2 & 2 \\ 0 & 2 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

So, the loop equations are

$$\begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

Solving these equations, $i_1 = 0.3\text{A}$, $i_2 = -1.1$ A

(c) Putting these values, the branch voltages are

$V_1 = 2 \times i_1 = 0.6$ V, $V_2 = 2 \times i_2 = -2.2$ V, $V_3 = -5$ V, $V_4 = -2 \times i_1 + 4 = 3.4$ V, $V_5 = 2.8$ V

Thus, the branch currents are

$$I_{AB} = \frac{3.4}{2} = 1.7\text{A}, I_{AD} = \frac{2.8}{2} = 1.4\text{A}, I_{AC} = \frac{5}{2} = 2.5\text{A}, I_{DB} = \frac{0.6}{2} = 0.3\text{A}, I_{DC} = \frac{2.2}{2} = 1.1\text{A}$$

So, the current supplied by the battery = (1.7 + 1.4 + 2.5 − 2) = 3.6 A

Problem 3.13 ***For the network shown in Fig. 3.30, draw the oriented graph and obtain the tie-set matrix. Use this matrix to calculate i.***

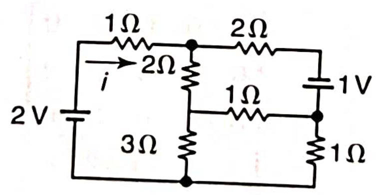


Fig. 3.30

Solution The oriented graph and any one tree are shown.

The tie-set matrix is given as

$$B_a = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix}$$

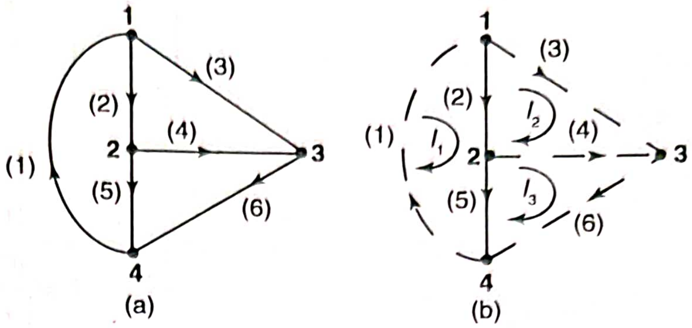


Fig. 3.31

The branch impedance matrix

$$Z_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore B_a Z_b = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 & 0 \\ 0 & -2 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 1 \end{bmatrix}$$

$$\therefore B_a Z_b B_a^T = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 & 0 \\ 0 & -2 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -2 & -3 \\ -2 & 5 & -1 \\ -3 & -1 & 5 \end{bmatrix}$$

Now,

$$-B_a V_s = -\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

So, the loop equations become

$$\begin{bmatrix} 6 & -2 & -3 \\ -2 & 5 & -1 \\ -3 & -1 & 5 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Solving for I_1

$$I_1 = \frac{\begin{vmatrix} 2 & -2 & -3 \\ 1 & 5 & -1 \\ 0 & -1 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & -2 & -3 \\ -2 & 5 & -1 \\ -3 & -1 & 5 \end{vmatrix}} = 0.91\,\text{A}$$

$\therefore\ I_1 = 0.91\ \text{A}$